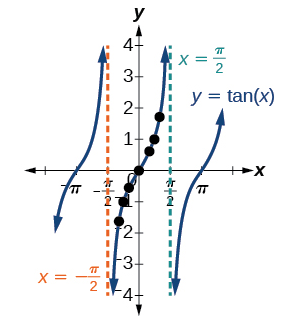
# Analyzing the Graph of

Just like with sine and cosine graphs, we can use a table of values to graph the tangent function and determine its characteristics. The period of the tangent function is because the graph repeats itself on intervals of where is a constant. If we graph the tangent function on to , we can see the behavior of the graph on one complete cycle.

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From the graph, we can see that there are places where the tangent function is undefined. The graph of the tangent has vertical asymptotes.



## Graphing One Period of a Tangent Function

The general form of the tangent function is

Features of the graph

• The stretching factor is .

• The period is .

• The domain is , where is an integer.

• The range is .

• The vertical asymptotes occur at , where is an odd integer.

Given the function , sketch the graph of one period.

Express the function given in the form

Identify the stretching/compressing factor, .

Identify and determine the period, .

Identify and determine the phase shift, .

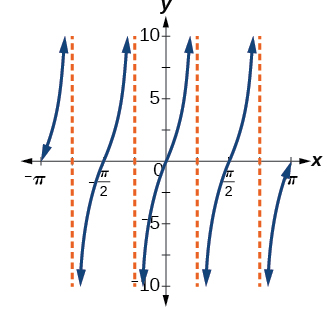
Draw the graph of shifted to the right by and up by .

Sketch the vertical asymptotes, which occur at , where is an odd integer.

Plot any three reference points and draw the graph through these points.

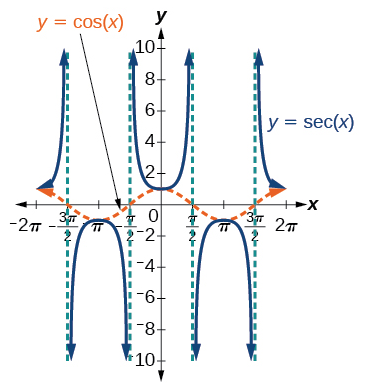
Examples

1. Sketch a graph of .
2. Graph one period of the function .
3. Find a formula for the function below.

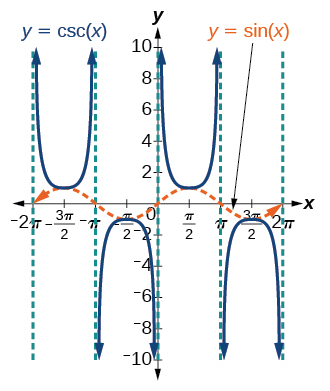


# Analyzing the Graph of and

Recall that . We can graph by observing the graph of the cosine function because these two functions are reciprocals of one another. For instance, the function is undefined when the cosine is (where the cosine graph crosses the -axis), leading to vertical asymptotes at etc. Where the graph of the cosine function increases, the graph of the secant function decreases (and vice versa).



Recall that . Similar to secant, we can graph by observing the graph of the sine function because these two functions are reciprocals of one another.



# Graphing Variations of and

For shifted, compressed, and/or stretched versions of the secant and cosecant functions, we can follow similar methods to those we used for the tangent function. The secant and cosecant functions can be described by

Features of the Graph

• The stretching/compressing factor is . (There is no amplitude.)

• The period is .

• The domain is where is an odd integer.

• The range is .

• The vertical asymptotes occur at , where is an odd integer.

• is an even function because cosine is an even function.

Features of the Graph

• The stretching/compressing factor is . (There is no amplitude.)

• The period is .

• The domain is where is an integer.

• The range is .

• The vertical asymptotes occur at , where is an integer.

• is an odd function because sine is an odd function.

Given the function , sketch the graph of one period.

Express the function given in the form

Identify the stretching/compressing factor, .

Identify and determine the period, .

Identify and determine the phase shift, .

Draw the graph of shifted to the right by and up by .

Sketch the vertical asymptotes, which occur at , where is an odd integer.

Given the function , sketch the graph of one period.

Express the function given in the form

Identify the stretching/compressing factor, .

Identify and determine the period, .

Identify and determine the phase shift, .

Draw the graph of shifted to the right by and up by .

Sketch the vertical asymptotes, which occur at , where is an integer.

Examples

1. Sketch a graph of .
2. Sketch a graph of .
3. Given the graph of shown below, sketch the graph of on the same set of axes.

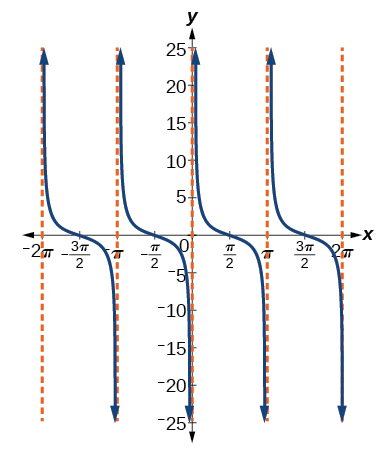
Chart, line chart

Description automatically generated

1. Graph one period of .
2. Sketch a graph of . What are the domain and range of this function?

# Analyzing the Graph of

Recall that . Just like with secant and cosecant functions, we see what the cotangent function is like by looking at its reciprocal, For instance, notice that the function is undefined when the tangent function is , leading to vertical asymptotes in the graph at etc.



# Graphing Variations of

We can transform the graph of the cotangent in much the same way as we did for tangent. The cotangent function can be described by a general equation.

Features of the Graph

• The stretching/compressing factor is . (There is no amplitude).

• The period is .

• The domain is , where is an integer.

• The range is .

• The vertical asymptotes occur at , where is an integer.

• is an odd function because it is the quotient of odd and even functions (sine and cosine respectively).

Given the function , sketch the graph of one period.

Express the function given in the form

Identify the stretching/compressing factor, .

Identify the period, .

Identify the phase shift, .

Draw the graph of shifted to the right by and up by .

Sketch the vertical asymptotes, which occur at , where is an integer.

Plot any three reference points and draw the graph through these points.

Examples

1. Determine the stretching factor, period, and phase shift of , and then sketch a graph.
2. Sketch a graph of one period of the function .

# Using the Graphs of Trigonometric Functions to Solve Real-World Problems

Many real-world scenarios represent periodic functions and may be modeled by trigonometric functions.

Examples

1. Suppose the function marks the distance in the movement of a light beam from the top of a police car across a wall where is the time in seconds and is the distance in feet from a point on the wall directly across from the police car.
   1. Find and interpret the stretching factor and period.
   2. Graph on the interval .
   3. Evaluate and discuss the function’s value at that input.
2. A laser rangefinder is locked on a comet approaching Earth. The distance , in kilometers, of the comet after days, for in the interval to days, is given by .
   1. Graph on the interval .
   2. Evaluate and interpret the information.
   3. What is the minimum distance between the comet and Earth? When does this occur? To which constant in the equation does this correspond?
   4. Find and discuss the meaning of any vertical asymptotes.